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Self-gravitational instability of a stratified plasma in the presence of Hall currents

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In this investigation we have studied the hydromagnetic stability of a self-gravitating, incompressible, inviscid, infinitely conducting plasma of variable density in the presence of the effect of Hall Currents. The prevailing magnetic field is assumed to be uniform and acting along the vertical. It is first shown that a variational principle characterizes the problem. Proper solutions have then been obtained, by the variational methods, for a semi-infinite plasma having a one-dimensional (exponential) density gradient along the vertical direction . The dispersion relation has been solved numerically and it is found that the growth rate increases with Hall currents, exhibiting thereby the destabilizing character of Hall currents on the dynamic stability of a stratified plasma.

1. INTRODUCTION

The effects of Hall currents are of importance in the dynamics of stellar atmospheres and interstellar matter and several other astrophysical situations. During the last few years the study of the effects of Hall currents on stability problems in hydromagnetics has attracted the attention of several authors. In particular, Talwar & Kalra (1967), Hosking (1968), Singh & Tandon (1969), Ariel (1970*a*, 1970*b*) have studied the influence of these effects on Rayleigh-Taylor instability problems and found that the Hall effect is in general destabilizing in inviscid, infinitely conducting liquids and can give rise to new unstable modes.

The hydromagnetic stability of a magnetized plasma of variable density is of considerable importance in a variety of astrophysical situations e.g., in theories of sunspot magnetic fields, heating of solar corona and the stability of stellar atmospheres in magnetic fields. Chandrasekhar (1961) has given a detailed account of the various investigations of this problem. Sundaram (1968) has investigated the dynamic stability of a self-gravitating plasma of variable density, in the absence of Hall currents, in which the prevalent magnetic field is uniform and along the vertical direction. The purpose of the present paper is to study the influence of the Hall effects on the self-gravitational instability of the above mentioned astrophysical plasmas of variable density. The importance of such a study in an astronomical context has also been pointed out earlier by Shafranov (1960) and Oganesyan (1960*a*, 1960*b*).

2. Perturbation Equations

We consider the motion of a self-gravitating horizontal strata of a heavy, inviscid, incompressible, infinitely conducting fluid of variable density in the presence of a uniform magnetic field.

In the equilibrium state the pressure p_0, density ρ_0, and the gravitational potential ϕ_0 are all taken to be function of vertical coordinate z. Assuming that the perturbations in pressure, density, velocity, magnetic field and gravitational potential are denoted respectively by δp, $\delta \rho$, $\boldsymbol{V}(u, v, w)$, $\boldsymbol{h}(h_x, h_y, h_z)$ and $\delta\phi$, the relevent equations of the problem are

$$\rho_0 \frac{\partial \boldsymbol{V}}{\partial t} = -\nabla \delta p + \frac{1}{4\pi}[(\nabla \times \boldsymbol{h}) \times \boldsymbol{H}] + \delta\rho\nabla\phi_0 + \rho_0 \nabla \delta\phi, \tag{1}$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{H}) - \frac{c}{4\pi N e}\{\nabla \times [(\nabla \times \boldsymbol{h}) \times \boldsymbol{H}]\}, \tag{2}$$

$$\frac{\partial}{\partial t}(\delta\rho) + (\boldsymbol{V}.\nabla)\rho_0 = 0, \tag{3}$$

$$\nabla^2 \delta\phi = -4\pi G \delta\rho, \tag{4}$$

$$\nabla.\boldsymbol{V} = 0, \qquad \nabla.\boldsymbol{h} = 0. \tag{5}$$

Here e and N denote respectively the charge and the number density of the particles of the medium, G is the gravitational constant and ∇^2 is the Laplacian operator.

We seek solutions of the above equations by analysing it in terms of normal modes whose dependence on space coordinates (x, y, z) and time t is of the form

$$F(z) \exp{(ik_x x + ik_y y + nt)}, \qquad \ldots \quad (6)$$

where $F(z)$ is some function of z, k_x and k_y are the wave numbers along x and y axes respectively and n (may be complex) is the growth rate of the perturbation.

Assuming that the prevailing magnetic field acts in the vertical direction i.e., $\boldsymbol{H} = (0, 0, H_0)$ and using expression (6) in the set of eqs. (1) to (5), we get

$$\rho_0 n u = -ik_x \delta p + \frac{H_0}{4\pi}(Dh_x - ik_x h_z) + \rho_0 ik_x \delta\phi, \qquad \ldots \quad (7)$$

$$\rho_0 n v = -ik_y \delta p + \frac{H_0}{4\pi}(Dh_y - ik_y h_z) + \rho_0 ik_y \delta\phi, \qquad \ldots \quad (8)$$

$$\rho_0 n w = -D\delta p + \rho_0 D\delta\phi - \frac{w D\rho_0 D\phi_0}{n}, \qquad \ldots \quad (9)$$

$$nh_x = H_0 Du + \left(\frac{cH_0}{4\pi Ne}\right) D(Dh_y - ik_y h_z), \qquad \ldots (10)$$

$$nh_y = H_0 Dv + \left(\frac{cH_0}{4\pi Ne}\right) D(ik_x h_z - Dh_x), \qquad \ldots (11)$$

$$nh_z = H_0 Dw + \left(\frac{cH_0}{4\pi Ne}\right) D(ik_y h_x - ik_x h_y), \qquad \ldots (12)$$

$$(D^2 - k^2)\delta\phi = \frac{4\pi G w D\rho_0}{n}, \qquad \ldots (13)$$

$$ik_x u + ik_y v + Dw = 0, \qquad \ldots (14)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \qquad \ldots (15)$$

where we have written $D = d/dz$ and $k^2 = k_x{}^2 + k_y{}^2$. In obtaining eqs. (9) and (13), eq. (3) has been used.

Eliminating some of the variables from the above equations we obtain the following four equations in w, h_z, ζ and ξ

$$\left\{n^2\rho_0 + \frac{(D\rho_0)(D\phi_0)}{n}\right\} w - \frac{n}{k^2} D(\rho_0 Dw) + \delta\phi D\rho_0 + \frac{H_0}{4\pi k^2}(D^2 - k^2)Dh_z = 0, \qquad \ldots (16)$$

$$nh_z = H_0 Dw - \frac{cH_0}{4\pi Ne} D\xi, \qquad \ldots (17)$$

$$n\rho_0\zeta = \frac{H_0}{4\pi} D\xi, \qquad \ldots (18)$$

$$n\xi = H_0 D\zeta + \frac{cH_0}{4\pi Ne}(D^2 - k^2)Dh_z, \qquad \ldots (19)$$

where ζ and ξ are respectively the vertical components of the vectors curl $\boldsymbol{V}$ and curl $\boldsymbol{h}$ and are given by

$$\zeta = ik_x v - ik_y u, \quad \xi = ik_x h_y - ik_y h_x \qquad \ldots (20)$$

Thus we have got a set of five equations (eqs. (13) and (16) to (19)) in five variables w, h_z, $\delta\phi$, ζ and ξ, which must be solved subject to the appropriate boundary conditions.

3. Boundary Conditions

In stability problems of this kind, the boundary conditions are not too simple and we have often to introduce appropriate approximations to make the analysis tractable.

We assume that the plasma under consideration is semi-infinite, i.e., it is infinitely extending along the two horizontal directions and is contained between two free boundaries, at $z = 0$ and $z = d$.

Since at the boundaries the normal component of velocity must vanish, therefore, we have

$$w = 0, \quad \text{at } z = 0 \text{ and } z = d. \qquad \dots (21)$$

The vanishing of the tangential stresses on the free boundaries gives rise to the conditions

$$D(\zeta) = D^2(w) = 0 \quad \text{at} \quad z = 0, \quad z = d. \qquad \dots (22)$$

Also on the free boundaries we have

$$\xi = D(h_z) = 0 \qquad \text{at} \quad z = 0, \quad z = d. \qquad \dots (23)$$

Finally the conditions that must be satisfied by $\delta\phi$ at the boundaries are

$$(D-k)\delta\phi = 0, \qquad \dots (24)$$

$$(D+k)\delta\phi = 0, \qquad \dots (25)$$

which are obtained by matching the solutions on the boundaries $z = 0$ and $z = d$, in view of the fact that ϕ and the normal component of grad ϕ are continuous across the surfaces.

It may be mentioned that the other types of boundary conditions are also possible in astrophysical situations. For mathematical simplicity we have assumed the boundaries to be free which are perhaps more appropriate for stellar atmospheres (Spiegel 1965). In any case it is hoped that these boundaries will at least give us an insight into the tendencies of the actual physical situations.

4. Variational Principle

Let there be solutions w_i, h_i, ζ_i, ξ_i, $\delta\phi_i$ belonging to the characteristic value n_i and w_j, h_j, ζ_j, ξ_j, $\delta\phi_j$ belonging to n_j, where we have dropped the suffix z on h for convenience.

Multiplying eq. (16) for the characteristic value n_i and w_j and integrating across the vertical extent L of the plasma we obtain

$$n_i \int_L [\rho_0 k^2 w_i w_j - D(\rho_0 D w_i) w_j] dz + \frac{k^2}{n_i} \int_L (D\phi_0)(D\rho_0) w_i w_j dz$$

$$+ \frac{H_0}{4\pi} \int_L [(D^2-k^2) D h_i] w_j dz + k^2 \int_L (\delta\phi_i)(D\rho_0) w_j dz = 0. \qquad (26)$$

Performing integration by parts and using the boundary conditions we obtain

$$n_i \int_L [\rho_0 k^2 w_i w_j - D(\rho_0 D w_i) w_j] dz = n_i \int_L \rho_0 (w_i w_j k^2 + D w_i D w_j) dz$$

$$= n_i I_1(i, j), \tag{27}$$

$$k^2 \int_L w_j \delta\phi_i D\rho_0 dz = -\frac{n_j k^2}{4\pi G} \int_L [D\delta\phi_i D\delta\phi_j + k^2 \delta\phi_i \delta\phi_j] dz$$

$$= -n_j I_2(i, j) \tag{28}$$

$$\frac{k^2}{n_i} \int_L D\phi_0 D\rho_0 w_i w_j dz = \frac{1}{n_i} I_3(i, j) \tag{29}$$

$$\frac{1}{4\pi} \int_L H_0[(D^2 - k^2) D h_i] w_j dz = n_j \int_L \rho_0 \zeta_i \zeta_j dz + \frac{n_j}{4\pi} \int_L (k^2 h_i h_j + D h_i D h_j) dz$$

$$+ \frac{n_i}{4\pi} \int_L \xi_i \xi_j dz,$$

$$= n_j I_4(i, j) + n_j I_5(i, j) + n_i I_6(i, j). \tag{30}$$

Combining eqs. (26) to (30), we get

$$n_i \left(I_1 + I_6 + \frac{I_3}{n_i^2} \right) + n_j (I_4 + I_5 - I_2) = 0. \tag{31}$$

Writing $i = j$ in the above equation and suppressing subscripts, we obtain

$$n \left(I_1 - I_2 + \frac{I_3}{n^2} + I_4 + I_5 + I_6 \right) = 0. \quad \ldots \tag{32}$$

In order to show that eq. (32) forms the basis for the variational formulation of the present problem, we consider the arbitrary variations δw, δh, $\delta\zeta$, $\delta\xi$ and $\delta(\delta\phi)$ in the corresponding physical quantities w, h, ζ, ξ and $\delta\phi$, compatible with the boundary conditions. To the first order, the consequent variations δn in n and δI_1 to δI_6 in the integrals I_1 to I_6 satisfy the relation

$$\delta n \left(I_1 - I_2 - \frac{I_3}{n^2} + I_4 + I_5 + I_6 \right) + n \left(\delta I_1 - \delta I_2 + \frac{\delta I_3}{n^2} + \delta I_4 + \delta I_5 + \delta I_6 \right) = 0, \quad \ldots \tag{33}$$

where

$$\tfrac{1}{2} \delta I_1 = \int_L [\rho_0 k^2 w - D(\rho_0 D w)] \delta w dz, \quad \ldots \tag{34}$$

$$\tfrac{1}{2} \delta I_2 = -\frac{k^2}{4\pi G} \int_L \delta(\delta\phi)(D^2 - k^2) \delta\phi dz \quad \ldots \tag{35}$$

$$\tfrac{1}{2}\delta I_3 = k^2 \int_L D\rho_0 D\phi_0 w\delta w dz, \qquad \dots (36)$$

$$\tfrac{1}{2}\delta I_4 = \int_L \rho_0 \zeta\delta\zeta dz, \qquad \dots (37)$$

$$\tfrac{1}{2}\delta I_5 = -\frac{1}{4\pi}\int_L \delta h(D^2-k^2)h dz, \qquad \dots (38)$$

$$\tfrac{1}{2}\delta I_6 = \frac{1}{4\pi}\int_L \xi\delta\xi dz. \qquad \dots (39)$$

In obtaining δI_1 to δI_6 suitable integrations by parts have been employed and appropriate boundary conditions have been used.

Also the variations δw, δh, $\delta\zeta$, $\delta\xi$ and $\delta(\delta\phi)$ are corrected by the relations

$$(D^2-k^2)\delta(\delta\phi) = \frac{4\pi G D\rho_0}{n}\delta w - \frac{\delta n}{n^2} 4\pi G w D\rho_0, \qquad \dots (40)$$

$$n\delta h + h\delta n = H_0 D(\delta w) - \frac{cH_0}{4\pi Ne} D(\delta\xi), \qquad \dots (41)$$

$$n\rho_0\delta\zeta + \rho_0\zeta\delta n = \frac{H_0}{4\pi} D(\delta\xi), \qquad \dots (42)$$

$$n\delta\xi + \xi\delta n = H_0 D(\delta\zeta) + \frac{cH_0}{4\pi Ne}(D^2-k^2)D(\delta h). \qquad \dots (43)$$

Combining eqs. (33) to (43) we get

$$\frac{\delta n}{2}\left(I_1 - I_2 - \frac{I_3}{n^2} + I_4 + I_5 + I_6 - \frac{I_7}{n} \right)$$

$$= -n\int_L \left[\rho_0 k^2 w - D(\rho_0 Dw) + \frac{H_0}{4\pi n}(D^2-k^2)Dh + \frac{k^2}{n^2} D\rho_0 D\phi_0 w + \frac{k^2}{n} D\rho_0 \delta\phi\right]\delta w dz,$$

where

$$I_7 = k^2 \int_L w\delta\phi D\rho_0 dz.$$

We observe that the quantity occurring as the coefficient of δw under the integral sign in the above equation vanishes if eq. (16) is satisfied. Hence a necessary and sufficient condition for δn to vanish to the first order for small arbitrary variations δw, δh, $\delta\zeta$, $\delta\xi$ and $\delta(\delta\phi)$ connected by the eqs. (40) to (43) and compatible with the boundary conditions is that w be a solution of the characteristic value problem. Thus a variational procedure for obtaining an approximate solution of the present problem is possible.

5. A Self-Gravitating Plasma of Varying Density

Now we make use of the existence of the variational principle to obtain the solution of the problem of a continuously stratified plasma of depth d in which the unperturbed density $\rho_0(z)$ varies exponentially i.e.,

$$\rho_0(z) = \rho_1 \exp(\beta z), \qquad \dots \quad (44)$$

where ρ_1 and β are constants. The density distribution $\rho_0(z)$, given by eq. (44), in conjunction with Poisson's equation which must be satisfied by ϕ_0, leads to the following distribution for ϕ_0

$$\phi_0(z) = \frac{4\pi G\rho_1}{\beta^2}\,(-e^{\beta z}+\beta z+1). \qquad \dots \quad (45)$$

Appropriate to the boundary conditions, let us take the trial solutions for w, h, ζ and ξ as

$$\begin{aligned} w(z) &= A \sin \alpha z, \\ h(z) &= B \cos \alpha z, \\ \zeta(z) &= E \cos \alpha z, \\ \xi(z) &= F \sin \alpha z, \end{aligned}$$

where A, B, E, F are constants and $\alpha = m\pi/d$, m being an integer. The corresponding trial solution for $\delta\phi$ will be

$$(D-k)\delta\phi = K\, e^{\beta z} \sin \alpha z.$$

It may be noted that the trial solution for $\delta\phi$ satisfies only one of the boundary conditions in the present problem, as in the earlier investigation of Sundaram (1968) in the absence of Hall currents.

Substituting the trial solutions for w, h, etc. in eq. (32), evaluating the various integrals contained there in, we obtain, after some straight forward but lengthy simplification, the dispersion relation

$$\sigma^6\left\{\left(\frac{e^{3m\pi a}-1}{3m\pi a}\right)\left(\frac{1}{1+\frac{9}{4}a^2}\right)(x^2+\tfrac{9}{2}a^2+1)\right\}$$

$$+\sigma^4\left\{4L^2\left(\frac{e^{3m\pi a}-1}{3m\pi a}\right)\left(\frac{1}{1+\frac{9}{4}a^2}\right)(1+x^2)(x^2+\tfrac{9}{2}a^2+1)\right.$$

$$+S\left[\left(\frac{e^{3m\pi a}-1}{3m\pi a}\right)\left(\frac{1}{1+\frac{9}{4}a^2}\right)x^2-\left(\frac{e^{4m\pi a}-1}{4m\pi a}\right)\left(\frac{a^2+x^2}{1+4a^2}+\frac{8a^3x}{(4a^2+x^2-4ax+1)^2}\right)\right]$$

$$\left.+\left(\frac{e^{2m\pi a}-1}{2m\pi a}\right)\left(\frac{1}{1+a^2}\right)(3x^2+2a^2x^2+4a^2+3)\right\}$$

$$+\sigma^2\Big\{L^4\Big(\frac{e^{3m\pi a}-1}{3m\pi a}\Big)\Big(\frac{1}{1+\frac{9}{4}a^2}\Big)(1+x^2)^2(x^2+\tfrac{9}{4}a^2+1)$$

$$+L^2\Big(\frac{e^{2m\pi a}-1}{2m\pi a}\Big)\Big(\frac{1}{1+a^2}\Big)\,(1+x^2)(3x^2+4a^2+3)$$

$$+2L^2S\Big[\Big(\frac{e^{3m\pi a}-1}{3m\pi a}\Big)\Big(\frac{1}{1+\frac{9}{4}a^2}\Big)\,(1+x^2)^2-\Big(\frac{e^{4m\pi a}-1}{4m\pi a}\Big)\,(1+x^2)$$

$$\times\Big(\frac{a^2+x^2}{1+4a^2}+\frac{8a^3x}{(4a^2+x^2-4ax+1)^2}\Big)\Big]+2S\Big[\Big(\frac{e^{2m\pi a}-1}{2m\pi a}\Big)\Big(\frac{1}{1+a^2}\Big)x^2$$

$$-\Big(\frac{e^{3m\pi a}-1}{3m\pi a}\Big)\times\Big(\frac{a^2+x^2}{1+\frac{9}{4}a^2}+\frac{6a^3x}{(4a^2+x^2-4ax+1)(a^2+x^2-2ax+1)}\Big)\Big]$$

$$+\Big(\frac{e^{m\pi a}-1}{m\pi a}\Big)\Big(\frac{1}{1+\frac{1}{4}a^2}\Big)\,(3x^2+a^2x^2+\tfrac{1}{2}a^2+3)\Big\}$$

$$+\Big\{L^4S\Big[\Big(\frac{e^{3m\pi a}-1}{3m\pi a}\Big)\Big(\frac{1}{1+\frac{9}{4}a^2}\Big)\,(1+x^2)x^2-\Big(\frac{e^{4m\pi a}-1}{4m\pi a}\Big)\,(1+x^2)^2$$

$$\times\Big(\frac{a^2+x^2}{1+4a^2}+\frac{8a^3x}{(4a^2+x^2-4ax+1)^2}\Big)\Big]+2L^2S\Big[\Big(\frac{e^{2m\pi a}-1}{2m\pi a}\Big)\Big(\frac{1}{1+a^2}\Big)$$

$$(1+x^2)x^2-\Big(\frac{e^{3m\pi a}-1}{3m\pi a}\Big)\,(1+x^2)\Big(\frac{a^2+x^2}{1+\frac{9}{4}a^2}+$$

$$\frac{6a^3x}{(4a^2+x^2-4ax+1)(a^2+x^2-2ax+1)}\Big)\Big]+S\Big[\Big(\frac{e^{m\pi a}-1}{m\pi a}\Big)\Big(\frac{1}{1+\frac{1}{4}a^2}\Big)x^2$$

$$-\Big(\frac{e^{2m\pi a}-1}{2m\pi a}\Big)\Big(\frac{a^2+x^2}{1+a^2}+\frac{4a^3x}{(a^2+x^2-2ax+1)^2}\Big)\Big]$$

$$+L^2\Big(\frac{e^{m\pi a}-1}{m\pi a}\Big)\Big(\frac{1}{1+\frac{1}{4}a^2}\Big)(1+x^2)^2\,(1+\tfrac{1}{2}a^2)+(1+x^2)\Big\}=0, \quad \ldots \quad (46)$$

where we have written

$$\sigma=\frac{n}{\alpha V_1},\quad x=\frac{k}{\alpha},\quad a=\frac{\beta}{\alpha},\quad V_1{}^2=\frac{H_0{}^2}{4\pi\rho_1},\quad S=\frac{4\pi G\rho_1}{\alpha^2V_1{}^2},\quad L=\frac{cH_0}{4\pi Ne\alpha^3V_1}.$$

Here L and S are the parameters measuring the effects of Hall currents and self-gravitation in terms of the magnetic field and V_1 is the Alfven velocity. The dispersion relation (46) is quite complex. We have performed numerical calculations to locate the roots of σ from eq. (46) against x, for several values of the parameters L, S, m and a.

Table 1. Values of growth rate (Multiplied by 10)

x	$L = 5$			$L = 10$			$L = 15$		
	$S = 5$	$S = 10$	$S = 20$	$S = 5$	$S = 10$	$S = 20$	$S = 5$	$S = 10$	$S = 20$
0.0	1.464758	2.224391	2.849553	2.124049	2.980571	3.980532	2.262505	3.181090	4.362862
0.2	2.391230	3.273985	4.066681	2.985320	4.129515	5.494386	3.118460	4.359702	5.972648
0.4	4.356250	5.728918	7.078736	4.994820	6.868951	9.174440	5.441484	7.172229	9.850576
0.6	6.871549	9.036243	11.33189	7.625314	10.50292	14.13309	7.798717	10.88842	15.01168
0.8	9.434871	12.48836	15.91265	10.29429	14.21288	19.24132	10.49232	14.66571	20.27852
1.0	11.73932	15.62907	20.15122	12.67139	17.52947	23.83364	12.88612	18.02862	24.98083
1.2	13.68808	18.30874	23.80461	14.65622	20.31056	27.70809	14.87821	20.83354	28.91754
1.4	15.29429	20.53957	26.87585	16.26587	22.57868	30.89358	16.48661	23.10522	32.12227
1.6	16.60779	22.38592	29.44757	17.55772	24.41130	33.49291	17.77094	24.92583	36.45321
1.8	17.68264	23.91756	31.61056	18.59394	25.89198	35.61614	18.79572	26.38413	36.79048
2.0	18.56628	25.19520	33.44300	19.42887	27.09373	37.35894	19.61722	27.55764	38.47864

In table 1 we plot the values of growth rate (positive real value of σ) against x for the Hall current parameter $L = 5,\ 10,\ 15$, taking $a = 0.1$, $m = 1$ and $S = 5,\ 10,\ 20$.

6. Conclusion

It is clearly seen from table 1 that the growth rate increases as the Hall current parameter L increases (for same x), showing thereby the destabilizing character of the Hall currents. This result is in agreement with the earlier observations of Hosking (1968), Ariel (1970*a*, 1970*b*) and others. It is also seen from the calculations that the influence of the effects of self-gravitations is also destabilizing, as σ increases with S.

We may thus conclude that the Hall currents have a destabilizing influence on the dynamic stability of an inviscid, incompressible and infinitely conducting self-gravitating plasma of varying density.

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